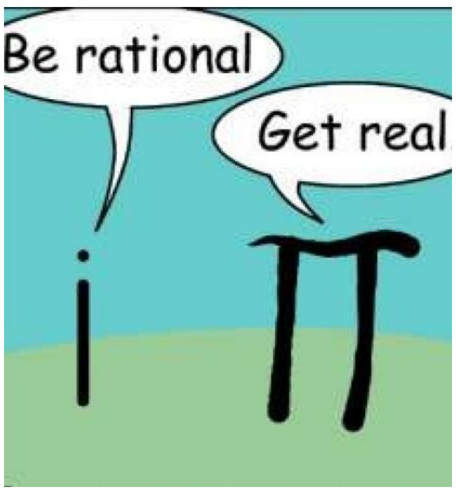
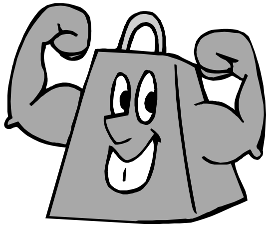
M314 Algebra II
Day 2

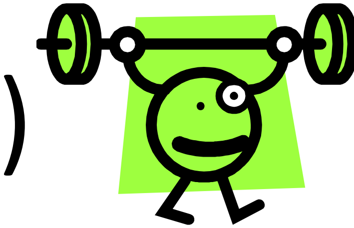
Section 7-7:
Imaginary and Complex Numbers



Presented by,
Mr. Kruczinski



Warm-up (#1 and #2)



Solve for x. Check for extraneous solutions:

1) $\sqrt{x} = 16$

$(\sqrt{x})^2 = (16)^2$

$x = 256$

Check:

$\sqrt{256} = 16$ ✓

2) $\sqrt[3]{x} - 2 = 4$

$(\sqrt[3]{x}) - 2 = 4$

$+2 \quad +2$

$(\sqrt[3]{x}) = 6$

$(\sqrt[3]{x})^3 = 6^3$

Check:

$(\sqrt[3]{216}) - 2 = 4$ ✓

Warm-up (#3)

3) $\sqrt{x-3} = -8$

$(\sqrt{x-3})^2 = (-8)^2$

$x - 3 = 64$

$x = 67$

Check:

$\sqrt{67-3} = -8$

$8 \neq -8$

$\boxed{\text{No solution}}$

Warm-up (#4)

4) $6 + \sqrt{8 - x} = x$

$$6 + (\sqrt{8-x}) = x$$

$$(\sqrt{8-x}) = x - 6$$

$$(\sqrt{8-x})^2 = (x-6)^2$$

$$8 - x = x^2 - 12x + 36$$

$$-8 + x \qquad -8 + x$$

$$0 = x^2 - 11x + 28$$

$$0 = (x-4)(x-7)$$

$$x - 4 = 0 \text{ or } x - 7 = 0$$

$$x = 4 \text{ or } x = 7$$

Check.....

$$\boxed{x = 7}$$

WORKSHEET #7

- For approximately 8 minutes work on problems : 1,3,7,8,9,10
- If you are done continue on with the worksheet

Imaginary Numbers:

In the set of real numbers, negative numbers do not have square roots

For example: $x^2 = -1$ does not have a real solution

However, imaginary #'s were invented so negative numbers could have square roots.

What was created was an "imaginary unit" called: i.

Powers of "i"

$i = \sqrt{-1} = i$

$i^2 = \underbrace{(\sqrt{-1})}_{i}\underbrace{(\sqrt{-1})}_{i} = (\sqrt{-1})^2 = -1$

$i^3 = i \cdot i \cdot i = i^2 \cdot i = (-1) \cdot i = -i$

$i^4 = i \cdot i \cdot i \cdot i = i^2 \cdot i^2 = (-1)(-1) = 1$

$i^5 = \underbrace{i \cdot i \cdot i \cdot i}\cdot i = i^4 \cdot i = 1 \cdot i = i$

$i^6 = \underbrace{i \cdot i \cdot i \cdot i}\cdot \underbrace{i \cdot i} = i^4 \cdot i^2 = 1 \cdot (-1) = -1$

Power of “i” [continued)

$$i^6 = i \cdot i \cdot i \cdot i \cdot i \cdot i = i^4 \cdot i^2 = 1 \cdot (-1) = -1$$

$$i^7 = i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = (1)(-1)(i) = -i$$

$$i^8 = i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = (1)(1) = 1$$

$$i^9 = i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = (1)(1)(i)$$

$$i^{10} = i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = (1)(1)(-1) = -1$$

$$i^{11} = i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = (1)(1)(-1)(i) = -i$$

Powers of “i” [continued]

There is a short-cut to this pattern…

- Look at the degree, then divide the degree by four.
- Example:

i^{24} look at degree

$\frac{24}{4} = 6$ groups of four

4 went in evenly into 24

$\therefore i^{24} = 1$

Powers of "i" [continued]

- Let us look at another example…

i^{53}

& 4 did not go in evenly

It went in 13.25 times

how many i's are left?

$53 - 4(13) = 53 - 52 = 1$

$\therefore$ One i is left

$\therefore i^{53} = i$

POWERS OF "i" [FINAL SLIDE]

i^{53}

$$\begin{array}{r} 13 \\ 4 \overline{)53} \\ -4\downarrow \\ \hline 13 \\ -12 \\ \hline 1 \end{array}$$

I have one i left over!

$\therefore i^{53} = i$

If I got a remainder of 2

- I would have two "i" left, which is $i^2 = -1$

If I got a remainder of 3

- I would have three "i" left, which is $i^3 = -i$

On your own attempt the following

Try

i^{67}

$$\begin{array}{r} 16 \\ 4\overline{)67} \\ -4 \\ \hline 27 \\ -24 \\ \hline 3 \end{array}$$

I have 3 i's remaining

$\therefore i^3 = \boxed{-i}$

EXAMPLES (#1 and #2)

EXAMPLES:

1.) $\sqrt{-5}$

$= \sqrt{-1 \cdot 5}$

$= \sqrt{-1} \cdot \sqrt{5}$

$= i \cdot \sqrt{5}$

$= \sqrt{5} \cdot i$

2.) $\sqrt{-36}$

$= \sqrt{-1 \cdot 36}$

$= \sqrt{-1} \cdot \sqrt{36}$

$= i\sqrt{36}$

$= i \cdot 6$

$= 6i$

EXAMPLES (#3 and #4)

3.) $\sqrt{-8}$

$= \sqrt{-1 \cdot 8}$

$= \sqrt{-1} \cdot \sqrt{8}$

$2 \quad 4$

$2 \quad 2$

$= i \cdot 2\sqrt{2}$

$\boxed{= 2\sqrt{2} \cdot i}$

4) $\sqrt{-7}$

$= \sqrt{-1 \cdot 7}$

$= \sqrt{-1} \cdot \sqrt{7}$

$= i\sqrt{7}$

$= \sqrt{7} \cdot i$

Definition

- Imaginary numbers are numbers expressed as bi . where…

$$\underbrace{b}_{\text{Real}} \cdot \underbrace{i}_{\text{Imaginary}}$$

ex. $\underbrace{6}_{\text{real}}\ \underbrace{i}_{\text{imaginary}}$

$\boxed{b=6}$ $b \neq 6i$

Multiplying Imaginary Numbers

- CAUTION!!! – Before you multiply you must first: convert to "bi" form.

1.) $4i \cdot 7$

$\underbrace{4i \cdot 7}_{\text{Multiply Real}}$

$4 \cdot 7 \cdot i$

$\boxed{28i}$

2.) $\sqrt{-3} \cdot 4$

$\sqrt{3}\, i \cdot 4$

$\boxed{4 \cdot \sqrt{3}\, i}$

Multiplying Imaginary Numbers [continued]

3.) $\sqrt{-3} \cdot \sqrt{-5}$

$\sqrt{3}\,i \cdot \sqrt{5} \cdot i$

$i \cdot i \cdot \sqrt{3} \cdot \sqrt{5}$

$i^2 \cdot \sqrt{3 \cdot 5}$

$-1 \cdot \sqrt{15}$

$\boxed{-\sqrt{15}}$

4) $\sqrt{-3} \cdot \sqrt{-3}$

$\sqrt{3}\,i \cdot \sqrt{3} \cdot i$

$\sqrt{3}\sqrt{3} \cdot i \cdot i$

$\sqrt{3 \cdot 3} \cdot i^2$

$\sqrt{9} \cdot (-1)$

$3 \cdot (-1)$

$\boxed{-3}$

<u>Complex Numbers:</u>

- are numbers in the form ___ $a+bi$ ___, where "a" and "b" are real #'s

$$\underbrace{a}_{\text{real}} + \underbrace{bi}_{\text{imaginary}}$$

- Example: $3+\sqrt{3}\,i$

$a = 3$

$b = \sqrt{3}$ $\quad b \neq \sqrt{3}\,i$

Write in “a+b*i*” Form

1.) $6i + 2i$

$\boxed{8i}$

$a = \underline{0}$

$b = \underline{8}$

2.) $\sqrt{4} + \sqrt{-5}$

$2 + \sqrt{5}\,i$

$a = \underline{2}$

$b = \underline{\sqrt{5}}$

Think of this as … “6x+2x=8x”

Write in "a+b*i*" form [Continued]

3.) $2\sqrt{3} + 3\sqrt{6}\,i$

$a = 2\sqrt{3}$

$b = 3\sqrt{6}$

4) $2 - \sqrt{-64}$

$2 - \sqrt{64}\,i$

$2 - 8i$

$a = 2 \quad b = -8$

Add or Subtract the following:

1.) $(1+i)+(2+3i)$

$= 1+i+2+3i$

$= 1+2+i+3i$

$= 3+4i$

$a=3 \quad b=4$

2.) $(8+i)-(3+2i)$

$8+i-3-2i$

$5-i$

$a=5 \quad b=-1$