# Law of Cosines -- An Exploration 



## Foreword:

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For this final project I have decided to do a deeper exploration on the Law of Cosines and ways to present this to a class of high school students. I have decided to create a Sketchpad Exploration that will help the students develop a reasonable hypothesis to lead them to the formula of the Law of Cosines. In particular, I am encouraging them to generalize the Pythagorean Theorem. The following is an outline of a lesson for introducing the Law of Cosines.

## Introductory Set:

I would start the lesson by telling a little story to my students...
Let's say you are Jay Leno and you are "Jay Walking" around the campus. You question every passerby about their mathematic knowledge. If you were asked them to state a major mathematic formula, they would probably answer " $a^{2}+b^{2}=c^{2}$."


These strangers may not be able to recall whose theorem it is and under what conditions it works. (To the class) What theorem is this and (as a hint) when does it apply to triangles?

The formula's purpose is to relate the two legs to the hypotenuse of a right triangle (see fig. 1). This is a very powerful result, but sadly we cannot apply this formula to all triangles. I would ask the class to think if there is a way to adapt this formula. Maybe we can attempt to add a "correction factor." i.e. $c^{2}=a^{2}+b^{2}+$ "correction factor." The following activity can be done as a demonstration or an individual/group exploration.

## Sketchpad Activity:

1. Go to http://jfkrucz.weebly.com/tech-tools.html and open Lawofcosines.gsp file.
2. The Sketchpad file contains a pre-made Triangle ABC with all the relevant objects have been measured.

- This has been premade to quicken the exploration and for the students to focus on the topic and not on using Sketchpad.
- Some of the action buttons should be eliminated for student version. Their purpose is to quicken this demonstration

3. The students can use Sketchpad's "calculate" feature to measure $a^{2}+b^{2}$ and $c^{2}$. Then they can calculate the "correction factor" which is $c^{2}-\left(a^{2}+b^{2}\right)$. Then the student can change the triangle by moving point A. By the construction sides "a" and "b" will remain constant. Questions to consider:

- What type of angle is Angle C when the correction factor is negative? When it is positive?

- How does this correction factor relate to Angle C?

4. Now we want to tabulate some data to develop an idea for the relationship between Angle C and the "Correction Factor." We want to do this to see if there is a function or constant that we can substitute the "Correction Factor." By "animating" point A, the table will record measures of Angle C's and the corresponding "Correction Factors."
5. After your data collection is complete plot the data (Angle $\mathbf{C}$ on the $\mathbf{X}$-axis and "Correction Factor" on the Y-axis). To better see the data, I would hide the table and the calculations. Questions to consider:

- What does this data tell us?
- Does the data suggest that there is a function that relates Angle C to the "Correction Factor"? What type of function does the plotted points look like?

6. To emphasize the fact that there is a function the teacher can plot the point (Angle C, "Correction Factor") and trace it by animating point $A$.
7. A class discussion of the fact that there is a function that will help adapt the Pythagorean Theorem to any triangle should take place after these discoveries. The teacher then can reveal the "correction function" to be 2ab* $\operatorname{cos(Angle~C)~}$

So the Law of Cosines states that:

$$
c^{2}=a^{2}+b^{2}-2 a b * \cos C
$$

**This makes the Pythagorean Theorem Stronger!**


## Back-up Plan:

If there was a technical difficulty or the school does not have the needed technology, I would devote more time to group discussion. I wasn't the students to postulate solutions. I would use the same introduction and attempt the students to engage in both critical thinking and problem solving. In groups, I would have the students brainstorm different ways that we can see if the "correction factor" is a function or some constant. I would ask some leading questions to help lead them in the right direction:

- How can we calculate the "correction factor" for any triangle?
- What variables should we be looking at and which variables should be held constant? I would eventually reveal to the students the formula and applications of the Law of Cosines.

